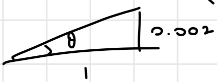
Pset 9 11/16/2024

King 7.1

a) $\sin\theta_n = n\frac{\lambda}{a}$. consider $n=1$. 10 fringes in 1.8cm $\Rightarrow$ 2 mm sep.

$a = 0.0003$



$\Rightarrow \sin(\arctan 0.002)\ 0.0003 = \lambda$

$= \boxed{6.00 \times 10^{-7} m}$

b) max. $x = L\theta = L\frac{n\lambda}{a}$. $n=2$, $\lambda = 6.33\times10^{-7}$, $L = 1.5$, $a = 0.0005$

$= 0.003798\ m$ $\times 2$ for symmetry so $\boxed{0.007596 m.}$

min: $x = (n + 1/2)\frac{L\lambda}{a}$

$= 0.0047475$ $\times 2$ $= \boxed{0.009495 m.}$

7.3 extra length travelled $= (n_{material} - n_{air})\,l = (n-1)\,l$

this accounts for a shift of $\Delta x = 15\lambda$

because the max is when path diff $= 0$.

$\Rightarrow l = \frac{15\lambda}{n-1} = \frac{15 \cdot 5\times10^{-7}}{1.6}$

$= \boxed{1.25 \times 10^{-5} m}$

7.7 a) $\frac{\lambda}{2}$ btw peaks. 1mm shift $=$ +4000 peaks

$\therefore \frac{\lambda}{2} = \frac{0.001}{4000} \Rightarrow \lambda = \frac{10^{-3}}{2\times10^{3}} = \boxed{5\times10^{-7} m.}$

b) $x = n\frac{\lambda_1}{2} = (n+1)\frac{\lambda_2}{2}$

$2x = n\lambda_1 = (n+1)\lambda_2$

$n = \frac{2x}{\lambda_1} \Rightarrow 2x = 2x\frac{\lambda_2}{\lambda_1} + \lambda_2$

$2x\lambda_1 = 2x\lambda_2 + \lambda_1\lambda_2$

$2x(\lambda_1 - \lambda_2) = \lambda_1\lambda_2$

$\Rightarrow x = \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)} = \boxed{2.89\times10^{-4} m}$

7.8 λ shrunk by factor of n.

$\lambda_0 = 633\,nm$. $\lambda = \lambda_0/n$

empty cell: # of wavelengths $= \frac{L}{\lambda_0} = \frac{2 \cdot l}{\lambda_0}$

full cell: # of " $= \frac{L}{\lambda} = \frac{2 \cdot l}{\lambda_0/n}$

$$\text{difference} = 90 = \frac{2ln}{\lambda_0} - \frac{2l}{\lambda_0} = \frac{2l}{\lambda_0}(n-1)$$

$$= \frac{2 \cdot 0.08}{6.33 \times 10^{-7}} \cdot (n-1)$$

$$\Rightarrow n = \frac{90 \cdot 6.33 \times 10^{-7}}{2 \cdot 0.08} + 1$$

$$= 1.000356$$

6-14

(a)

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) = y(x) = Ax(L-x)$$

$$\sum_{n=1}^{\infty} B_n \int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) dx = \int_0^L Ax(L-x)\sin\left(\frac{m\pi x}{L}\right) dx$$

$$\frac{L}{2} B_n = \int_0^L Ax(L-x)\sin\left(\frac{n\pi x}{L}\right) dx$$

$$B_n = \frac{2}{L}\int_0^L \left(ALx - Ax^2\right)\sin\left(\frac{n\pi x}{L}\right) dx$$

Wolfram-Alpha

$$= -\frac{2AL^2\left(n\pi\sin(n\pi) + 2\cos(n\pi) - 2\right)}{n^3\pi^3}$$

$$= \begin{cases} -\dfrac{2AL^2(0-2-2)}{n^3\pi^3} & \text{for odd } n \\[2ex] -\dfrac{2AL^2(0+2-2)}{n^3\pi^3} & \text{for even } n \end{cases}$$

$$= \begin{cases} \dfrac{8AL^2}{n^3\pi^3} & \text{for odd } n \\[2ex] 0 & \text{for even } n \end{cases}$$

(b) $B_1 = A$, $B_n = 0$ $n \neq 1$

(c)

$$B_n = \begin{cases} \dfrac{(-1)^{\frac{(n+1)}{2}}\, 4A}{\pi (n^2-4)} & n \text{ odd} \\[2ex] \dfrac{A}{2} & n = 2 \\[1ex] 0 & n \text{ even},\ n \neq 2 \end{cases}$$

(b)

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) \quad \text{with } B_n \text{ as follows:}$$

$$B_n = \frac{2}{L}\int_0^L A\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \begin{cases} A & n = 1 \\ 0 & n \neq 1 \end{cases}$$

(c)

$$\sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) \quad \text{with } B_n \text{ as follows:}$$

$$B_n = \frac{2}{L}\int_0^{\frac{L}{2}} A\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right) dx + \frac{2}{L}\int_{\frac{L}{2}}^{L} 0\,\sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \begin{cases} \dfrac{2}{L}\cdot\dfrac{AL}{4} & n = 2 \\[2ex] -\dfrac{A\left((n-2)\sin\left(\frac{\pi n + 2\pi}{2}\right) + (-n-2)\sin\left(\frac{\pi n - 2\pi}{2}\right)\right)}{\pi(n^2-4)} & n \neq 2 \end{cases}$$

Wolfram-Alpha

$$= \begin{cases} \dfrac{A}{2} & n = 2 \\[2ex] \dfrac{-A\left(-(n-2) - (-n-2)\right)}{\pi(n^2-4)} & n = 1+4k \quad \text{where } k = 0, \pm 1, \pm 2, \ldots \\[2ex] \dfrac{-A\left((n-2) + (-n-2)\right)}{\pi(n^2-4)} & n = 1+2k \quad \text{where } k = 0, \pm 1, \pm 2, \ldots \\[2ex] 0 & \text{for even } n,\ n \neq 2 \end{cases}$$

$$= \begin{cases} \dfrac{A}{2} & n = 2 \\[2ex] -\dfrac{4A}{\pi(n^2-4)} & n = 1+4k \quad \text{where } k = 0, \pm 1, \pm 2, \ldots \\[2ex] \dfrac{4A}{\pi(n^2-4)} & n = 1+2k \quad \text{where } k = 0, \pm 1, \pm 2, \ldots \\[2ex] 0 & \text{for even } n,\ n \neq 2 \end{cases}$$

$$= \begin{cases} \dfrac{A}{2} & n = 2 \\[2ex] 0 & \text{for even } n \\[2ex] \dfrac{(-1)^{\frac{n+1}{2}}\, 4A}{\pi(n^2-4)} & \text{for odd } n \end{cases}$$

Pset 10 12/7/2024

King 8.1

beat freq $= f_2 - f_1 = 462\,\text{MHz}$

$$\frac{v}{\lambda_2} - \frac{v}{\lambda_1} = v\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = 462\ \text{MHz}$$

$v = c$, $\lambda_1 = 766.49110\,\text{nm}$. $\Rightarrow \boxed{\lambda_2 = 766.49020\ \text{nm}.}$

8.4

a) i) $v \propto \lambda^{-1} \Rightarrow v = \frac{\omega}{k} = \frac{a}{\lambda}$, $a = \text{const.}$

$$\omega = \frac{ka}{\lambda} = \frac{k^2 a}{2\pi}$$

$$\frac{d\omega}{dk} = \frac{ka}{\pi} = \frac{2a}{\lambda}. \quad \boxed{v_g = 2v_p}$$

ii) $v \propto \lambda^{-1/2} \Rightarrow v = \frac{\omega}{k} = \frac{a}{\sqrt{\lambda}}$

$$\omega = \frac{ka}{\sqrt{\lambda}} = \frac{k^{3/2}a}{\sqrt{2\pi}}$$

$$\frac{d\omega}{dk} = \frac{3}{2}\frac{k^{1/2}a}{\sqrt{2\pi}} = \frac{3}{2}\frac{a}{\lambda}. \quad \boxed{v_g = \frac{3}{2}v_p}$$

b) $v_p = \frac{\omega}{k} = c \quad v_g = \frac{d\omega}{dk} = c.$ cosmic speed limit ✓

c) $\varepsilon_r = \frac{c^2}{v^2} = 1 - \frac{\omega_0^2}{\omega^2}$

$$v^2 = \frac{\omega^2}{k^2} \Rightarrow \frac{c^2k^2}{\cancel{\omega^2}} = \frac{\omega^2 - \omega_0^2}{\cancel{\omega^2}} \quad \omega \neq 0.$$

then, $\boxed{\omega^2 = \omega_0^2 + c^2k^2}$, as required.

8.8

a) $y = A\cos(\omega t - kx)$. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 y$, $\frac{\partial^2 y}{\partial x^2} = -k^2 y$.

$$-\omega^2 y = -k^2\frac{T}{\mu}y - \alpha y = -\left(k^2\frac{T}{\mu} + \alpha\right)y \quad \Rightarrow \boxed{\omega^2 = k^2\frac{T}{\mu} + \alpha}$$

b) set $k = 0 \Rightarrow \boxed{\omega_{min} = \sqrt{\alpha}}$

c) $v_p = \frac{\omega}{k} = \sqrt{\frac{T}{\mu} + \frac{\alpha}{k^2}}$. $\frac{d\omega}{dk} = \cancel{\frac{1}{2}}\cancel{2}\frac{T}{\mu}k \cdot \frac{1}{\sqrt{k^2\frac{T}{\mu} + \alpha}} = \frac{T}{\mu} \cdot \frac{\cancel{k}}{\cancel{k}\sqrt{\frac{T}{\mu} + \frac{\alpha}{k^2}}}$

$$\Rightarrow \boxed{v_p v_g = \frac{T}{\mu}.}$$

8.9

$\Delta t \Delta \omega \approx 2\pi$

$$\Delta f = \frac{\Delta \omega}{2\pi} \approx \frac{1}{\Delta t} = \frac{1}{5\times 10^{-8}s} = \boxed{20 MHz}$$

French 7.17

a) rearrange w/ $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

$$\alpha+\beta = 5x - 10t$$
$$\alpha-\beta = 4x - 9t$$
$$\alpha = \tfrac{1}{2}(9x - 19t),\ \beta = \tfrac{1}{2}(x-t)$$

so $\boxed{y_{1+2}} = 2A\sin\left(\frac{9x-19t}{2}\right)\cos\left(\frac{x-t}{2}\right)$

b) $v_g = \frac{\Delta\omega}{\Delta k} = \frac{9-10}{4-5} = \boxed{1 m/s}$

(discrete ω)

c) $\cos\left(\frac{x-t}{2}\right)$. choose $t=0$ Then $\cos = 0$ when $\frac{x}{2} = (2n+1)\frac{\pi}{2}$, $n = 0,1,2,\dots$

$x = (2n+1)\pi \Rightarrow \boxed{2\pi \text{ m separation}}$

7.18

a) $v_p = \frac{\omega}{k} = \left(\frac{2\pi S}{\rho\lambda}\right)^{1/2} \Rightarrow \omega = \left(\frac{k^3 S}{\rho}\right)^{1/2}$

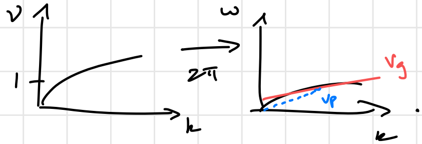
$$\frac{d\omega}{dk} = \frac{3}{2}\left(\frac{kS}{\rho}\right)^{1/2} = \frac{3}{2}\left(\frac{2\pi S}{\rho\lambda}\right)^{1/2} = \boxed{\frac{3}{2}v_p}$$

b) the groups seem to travel faster than the individual ripples, so the ripples will seem to lag and disappear into the back of the group.

c) $\underbrace{\frac{k_2-k_1}{2}}_{\text{modulation}} \cdot \underset{\uparrow\ \text{beat}}{2} = k_2 - k_1 = \cancel{2\pi}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = \cancel{2\pi}\left(\frac{1}{\lambda_{beat}}\right)$

$$\Rightarrow \lambda_{beat} = \frac{1}{\left|\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right|} \approx \boxed{50 cm}$$

7.19 $\omega = 2\pi\nu$



From textbook, we know $\boxed{v_g < v_p}$ for all values.